Likelihood- misfit issues related to the nature of data

$$P(\mathbf{d},\mathbf{m}) = \frac{1}{\left(2\pi\sigma^2\right)^{N/2}} \exp\left[-\underbrace{\left|\mathbf{d}-\mathbf{d}(\mathbf{m})\right|^2 / 2\sigma^2}_{\text{misfit } E(\mathbf{m}) \text{ (least squares)}}\right]$$

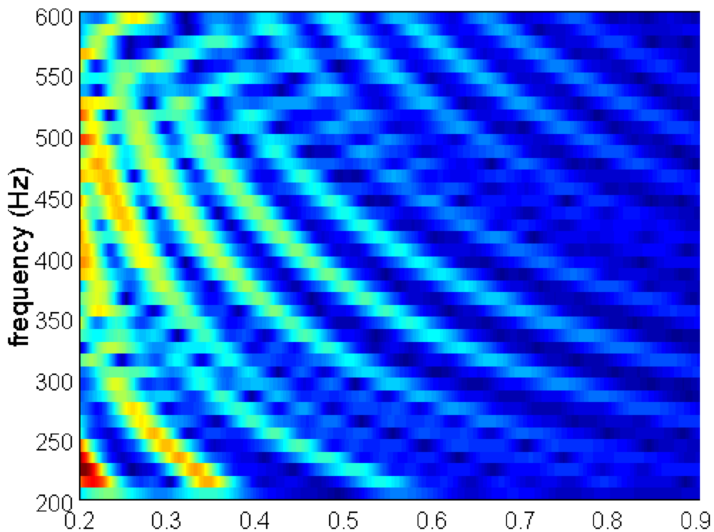
d is typically full field or reflection coefficients.

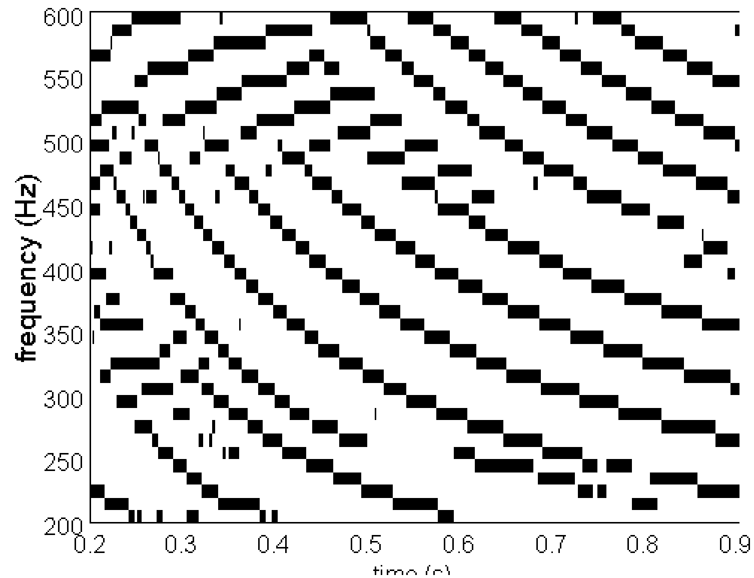
- Alternatively, inversion can be often performed with arrival time measurements (**d** now consists of arrival times) via linearization or global optimization approaches. Arrival times are assumed to vary in pre-specified ways. Errors in arrival times translate into uncertainty in inversion results.

- Arrival time errors are typically assumed to be zero-mean normally distributed, leading to the least squares error.

- Normality is not necessarily the case; multimodality and skewness are ubiquitous when we extract arrival times to be used for inversion.

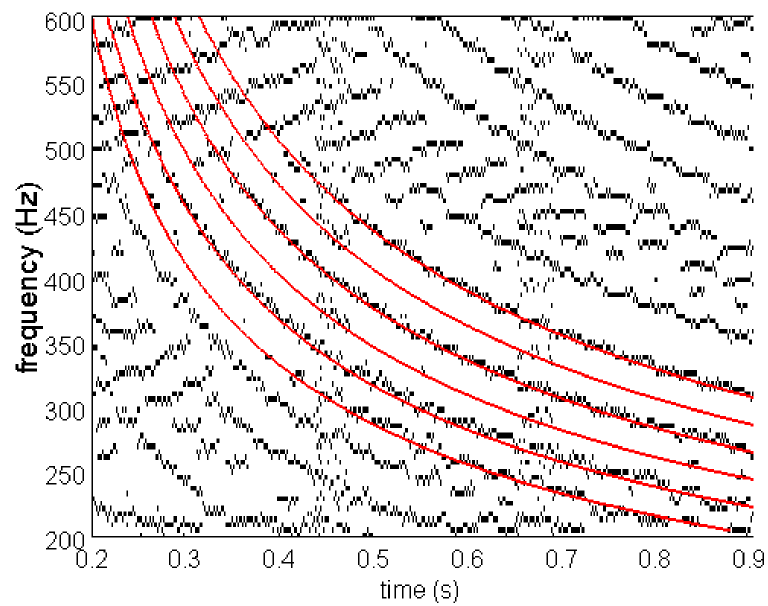
Identifying stochastic nature of data:
Group velocity estimation with uncertainty tracking



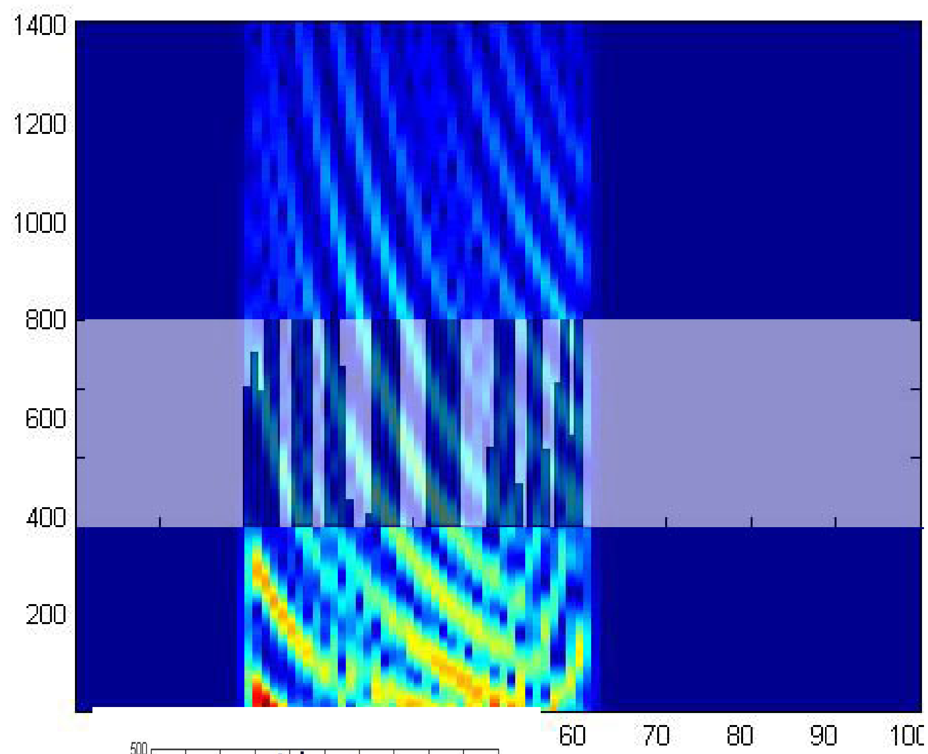


Identifying modal arrivals with sequential filtering to model dispersion:

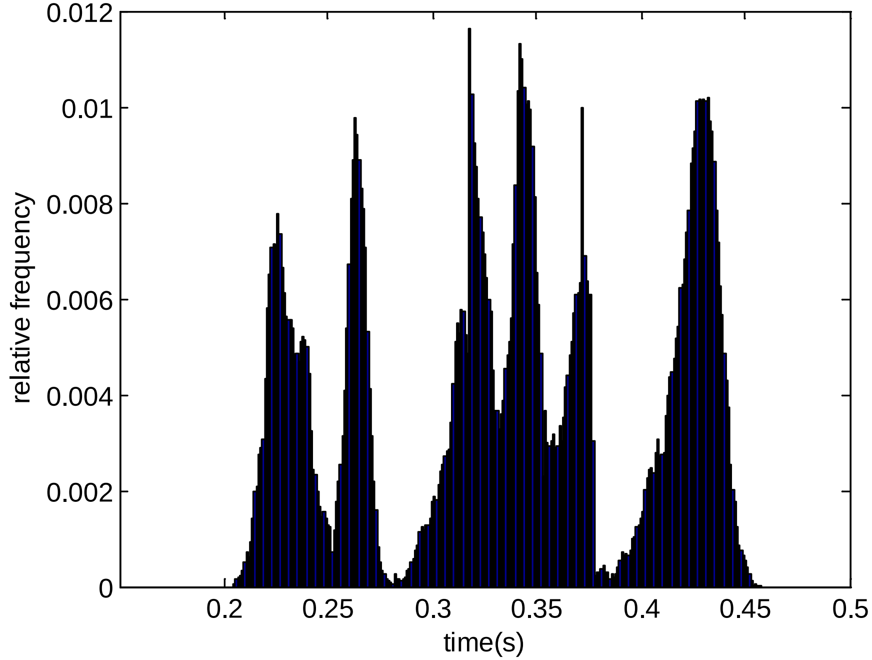
Frequency-arrival time pairs are associated with probabilities.

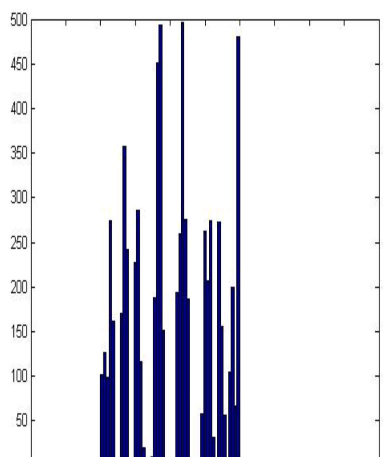


PDF estimation for modal frequencies-arrival times



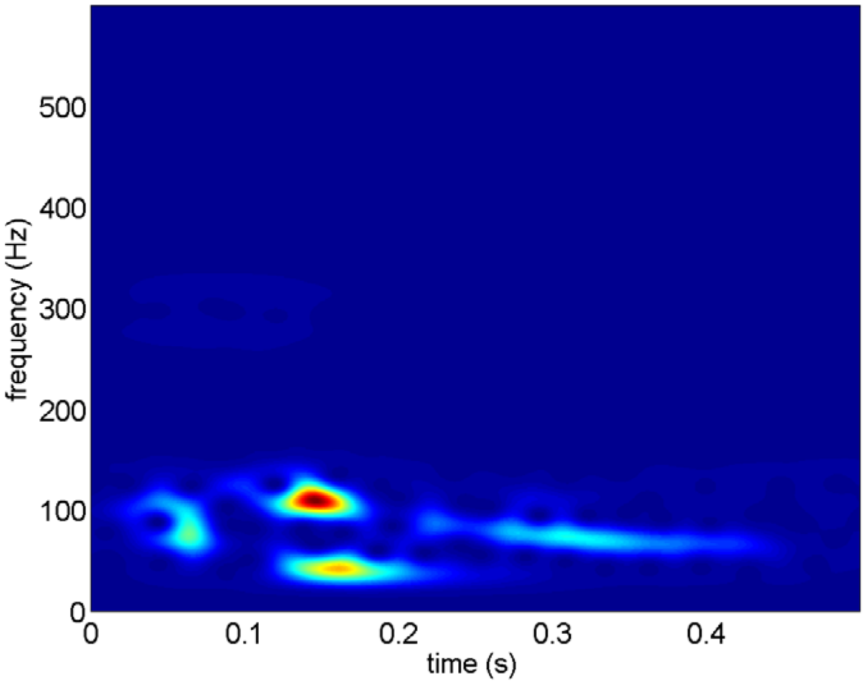
PDFs of arrival times for a specific modal frequency: deviation from normality is evident as well as level of belief in each mode



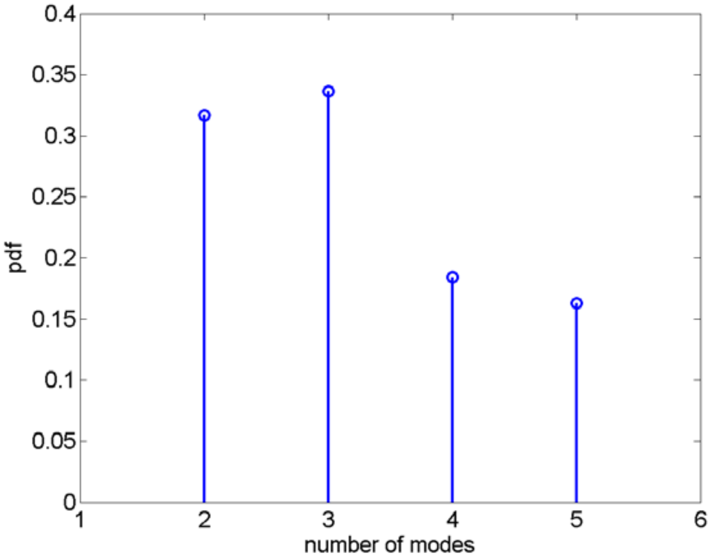


PDFs of modal frequencies arriving at a specific time

Shallow Water-06 – Mode identification with BIC



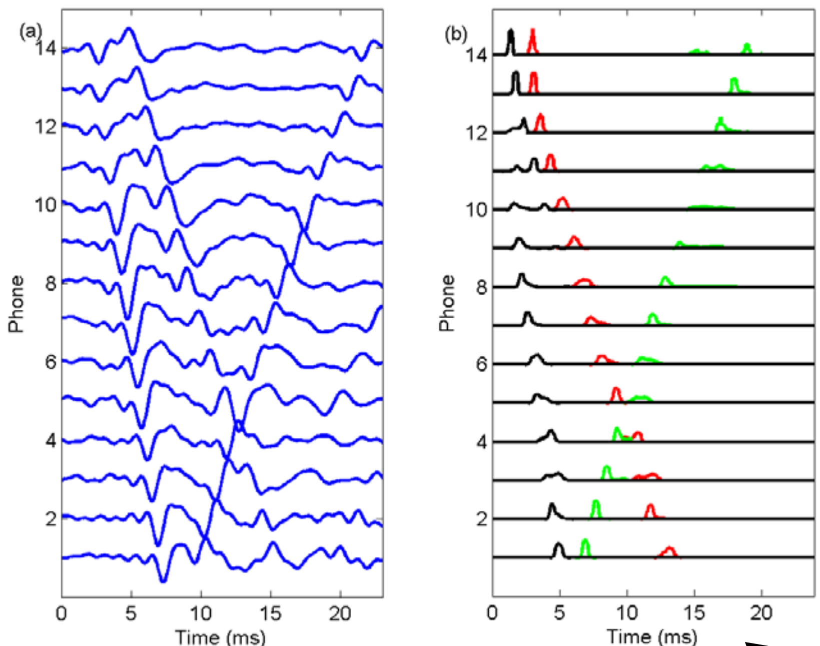
Probability mass density for number of modes being present

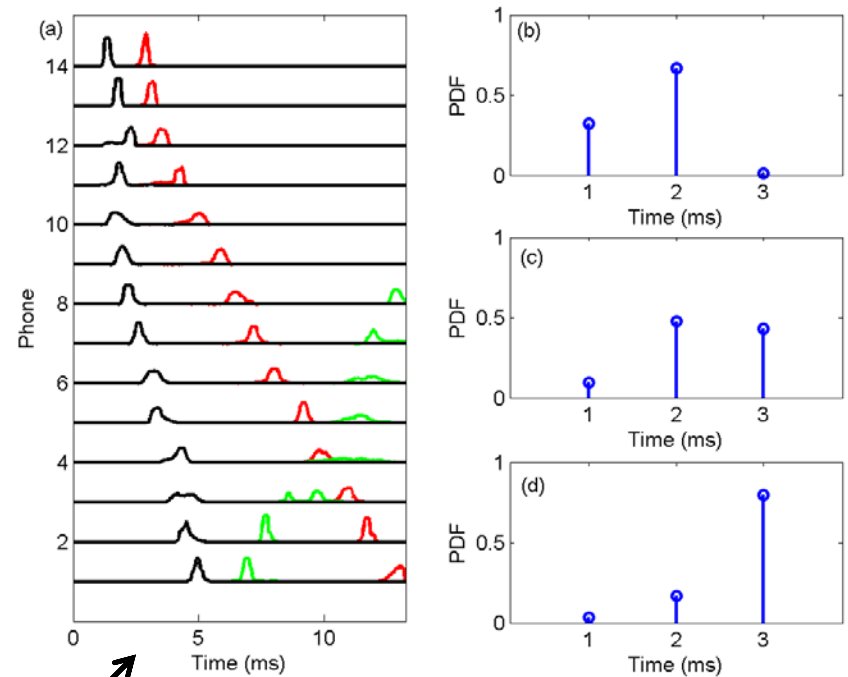


The number of modes present in the data is unclear.

SW06 – inversion with arrival times (short range) - BIC

Model Order PMF





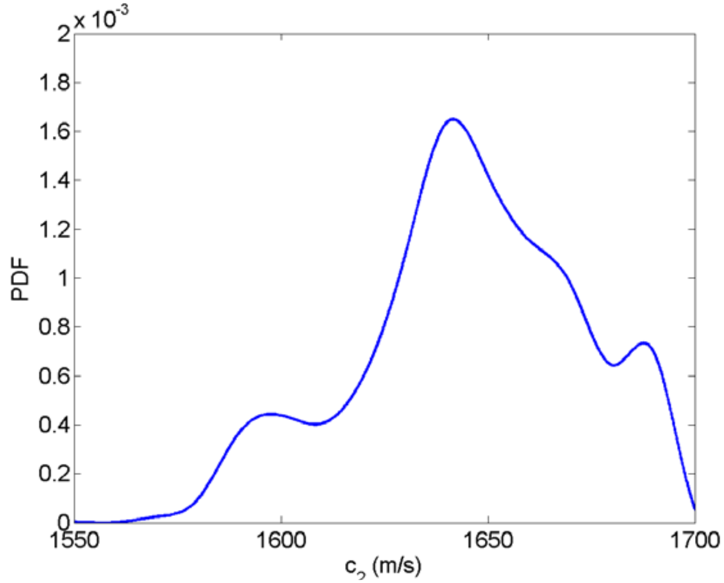
lfm pulses, 100-900 Hz

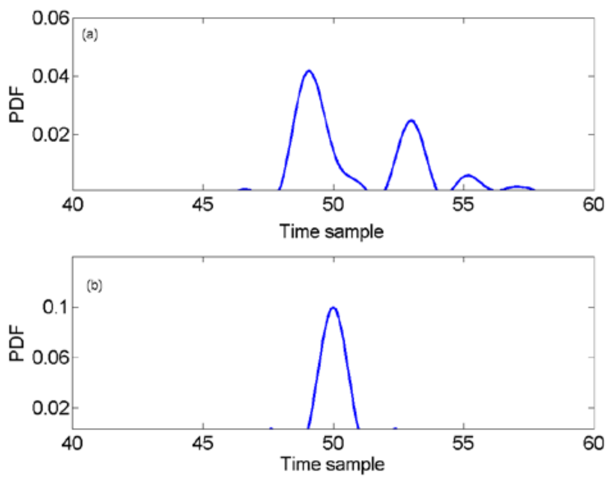
Arrival time PDFs

Phones 9, 8, and 7

Inversion for sediment sound speed – *same prior in both cases*

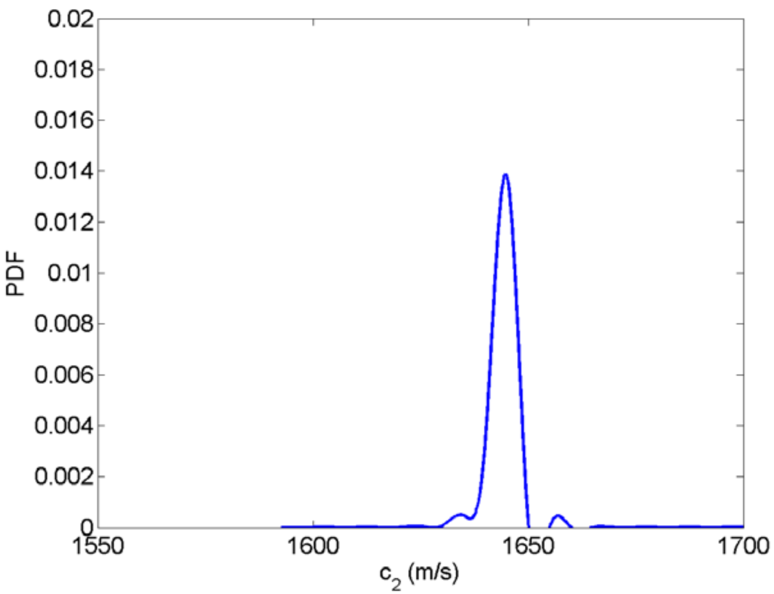
Propagating the time PDF





Arrival times used: before and after

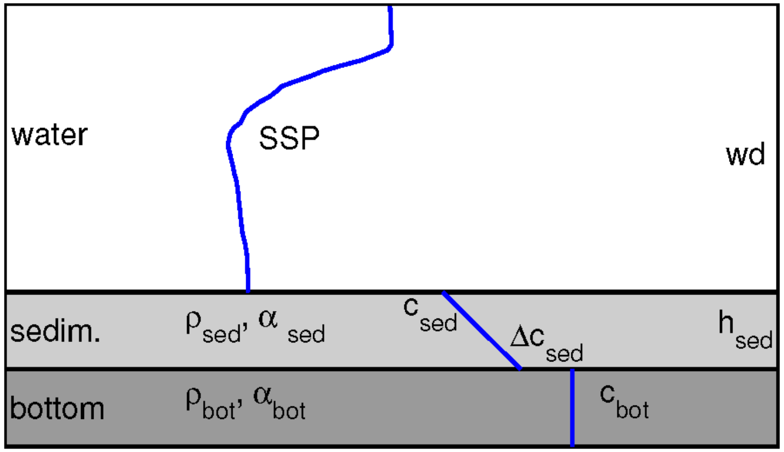
after filtering data using measurements from different phones



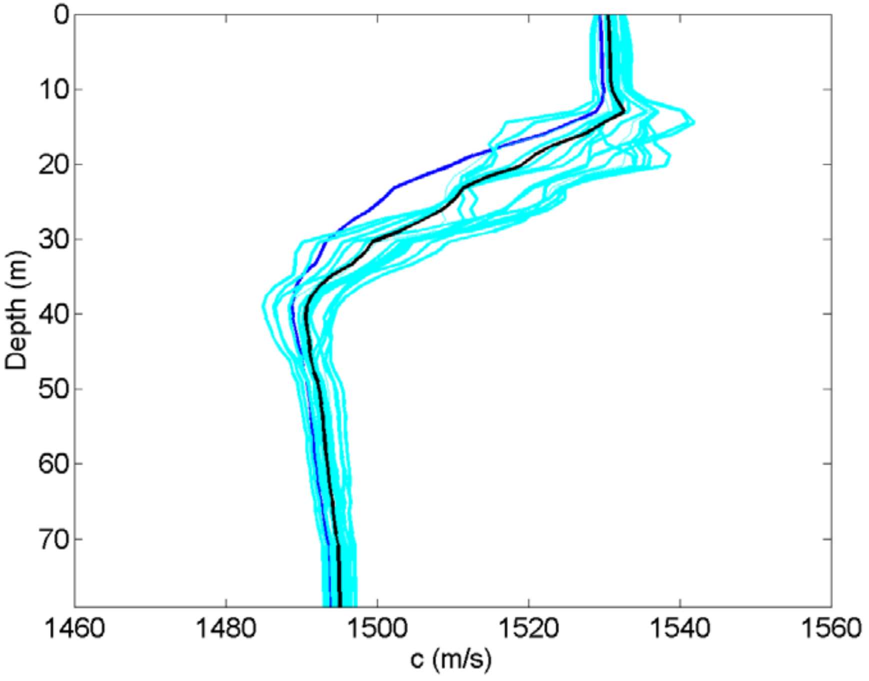
The same acoustic data and prior information are used, with different PDFs resulting from each inversion.

Inversion – SW06 with filtered data

Transmission of lfm pulses (100 – 900 Hz) in two frequency ranges (match filtering to obtain impulse response)



Source range: 230 m
Source depth: 25 m
Ocean depth: 80 m
Sound speed modeling through EOFs
Sediment thickness: 24 m
Sound speed in sediment: ~ 1600 m/s
16 receiving phones (low SNR in top two)

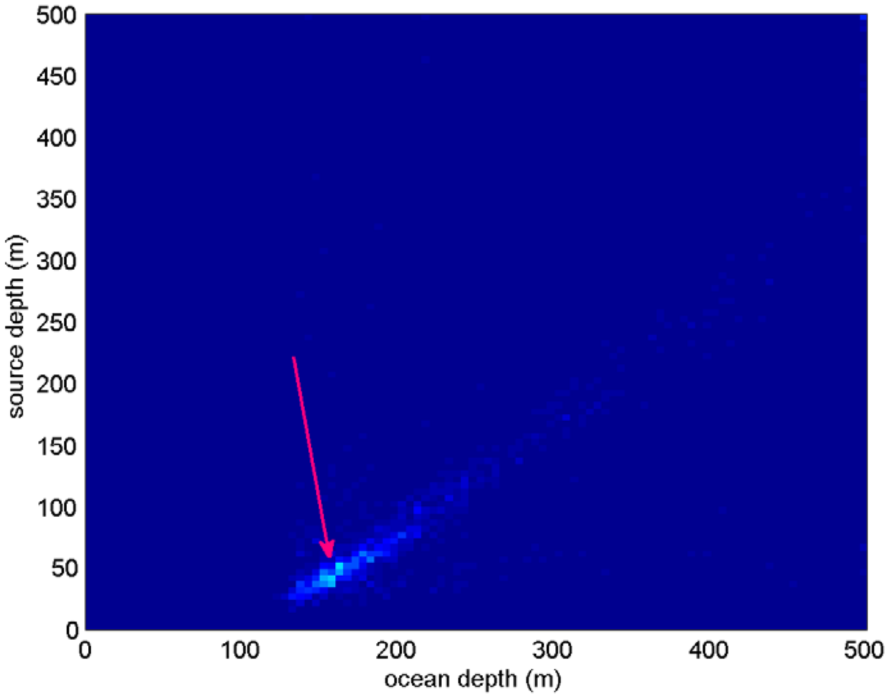


EOF coefficient PDFs were calculated.

Blue: mean ssp
Black: MAP ssp
Cyan : particles

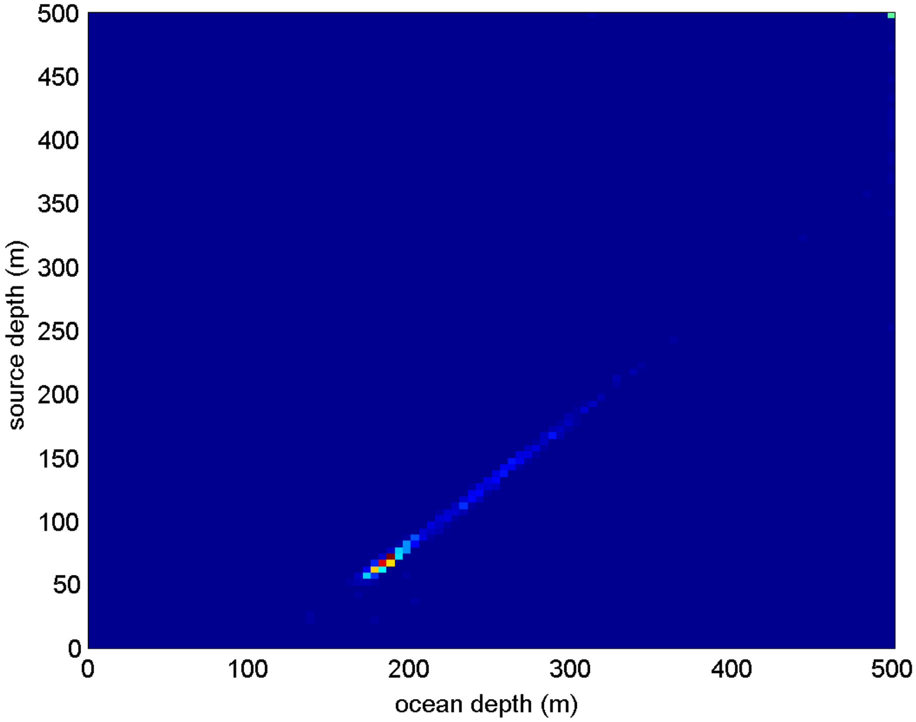
Localization and bathymetry – same priors (HS data)

Marginal posterior density functions for ocean – source depth

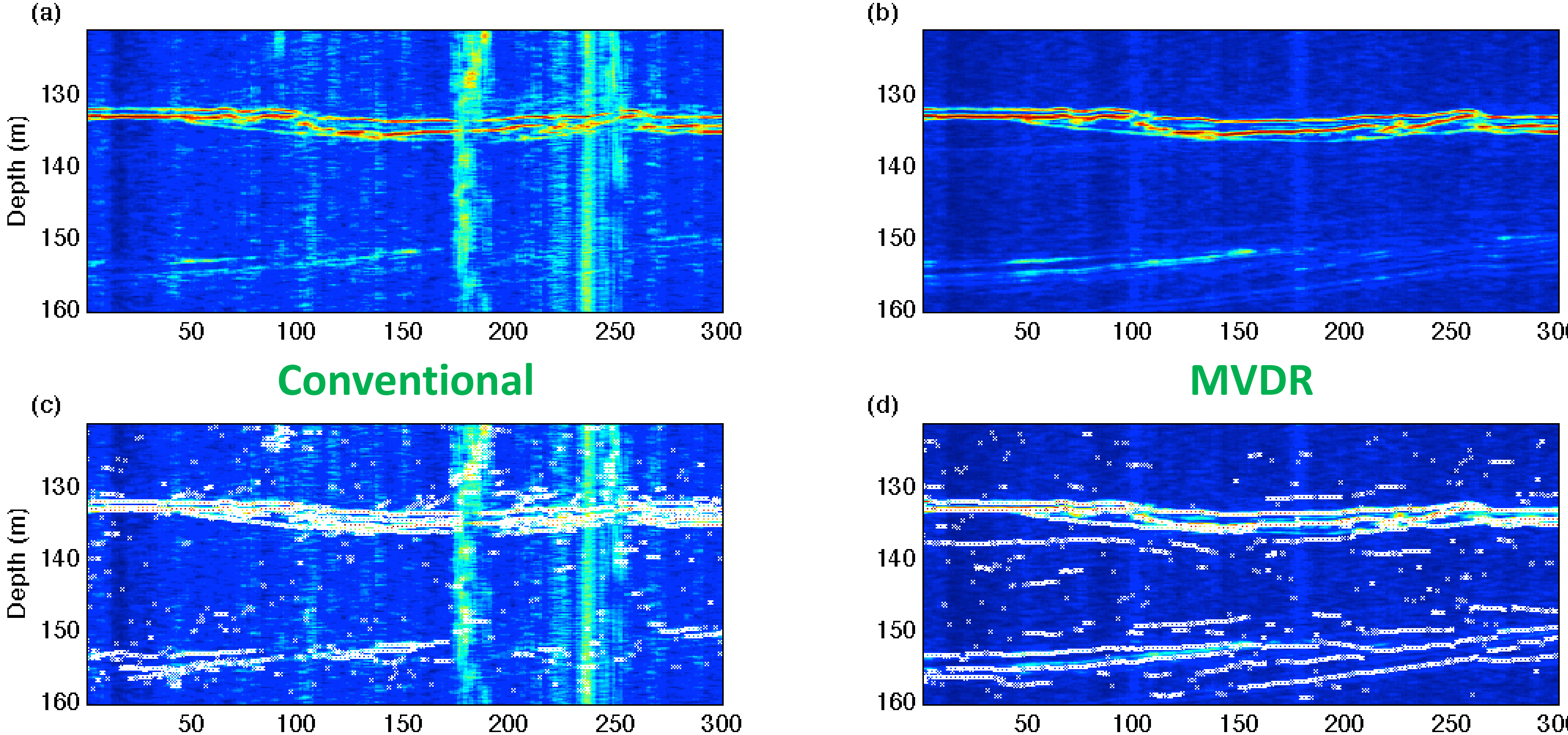


Data with conventional assumptions on arrival time uncertainty

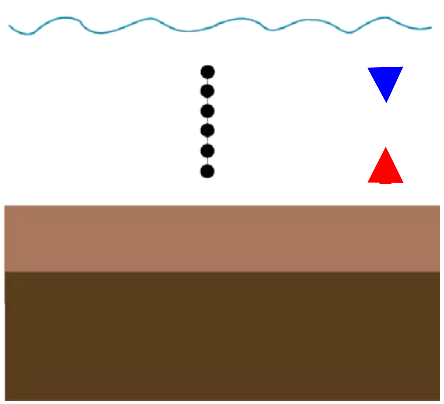
Filtered data



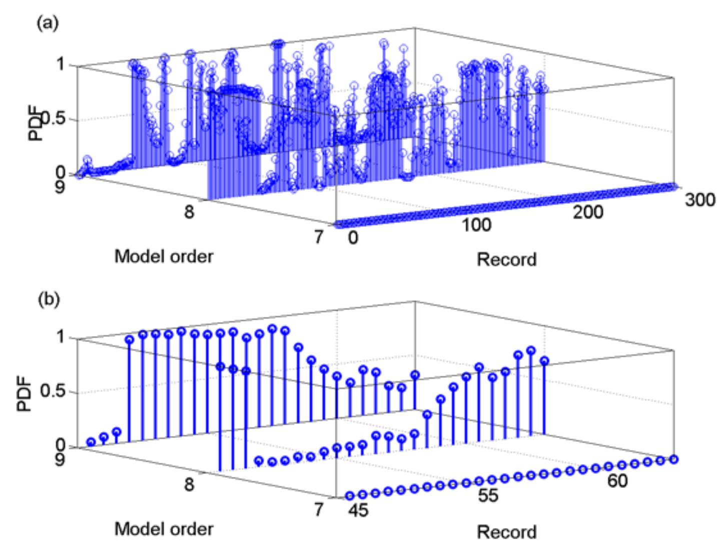
N-layer model - Passive fathometer tracking



Record (as the array drifts)

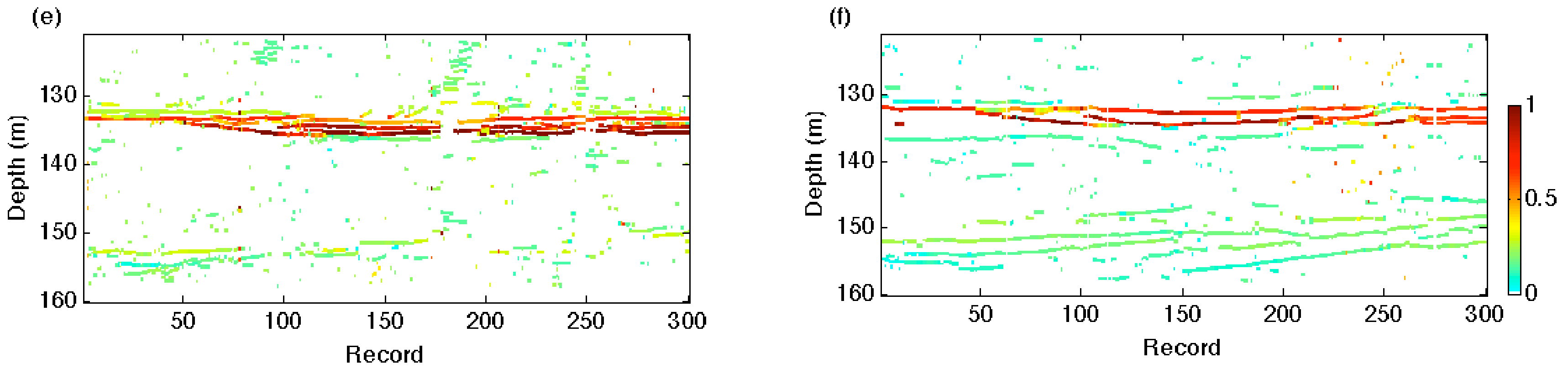


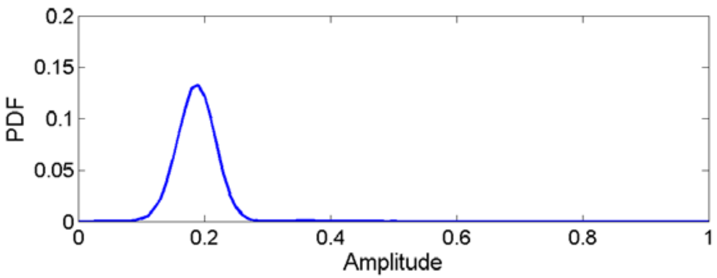
Model order estimation (number of reflectors) using a multiple model particle filter.

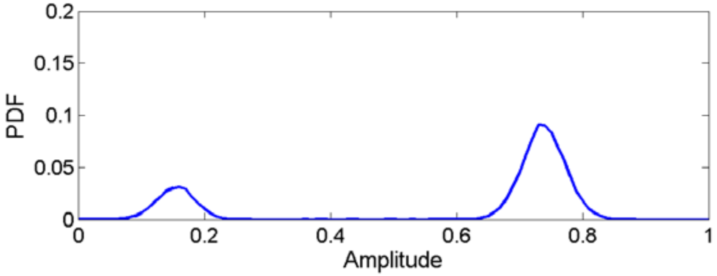


N-layer model – Passive fathometer tracking – amplitude estimation

Amplitude MAP estimates of reflections vs. record



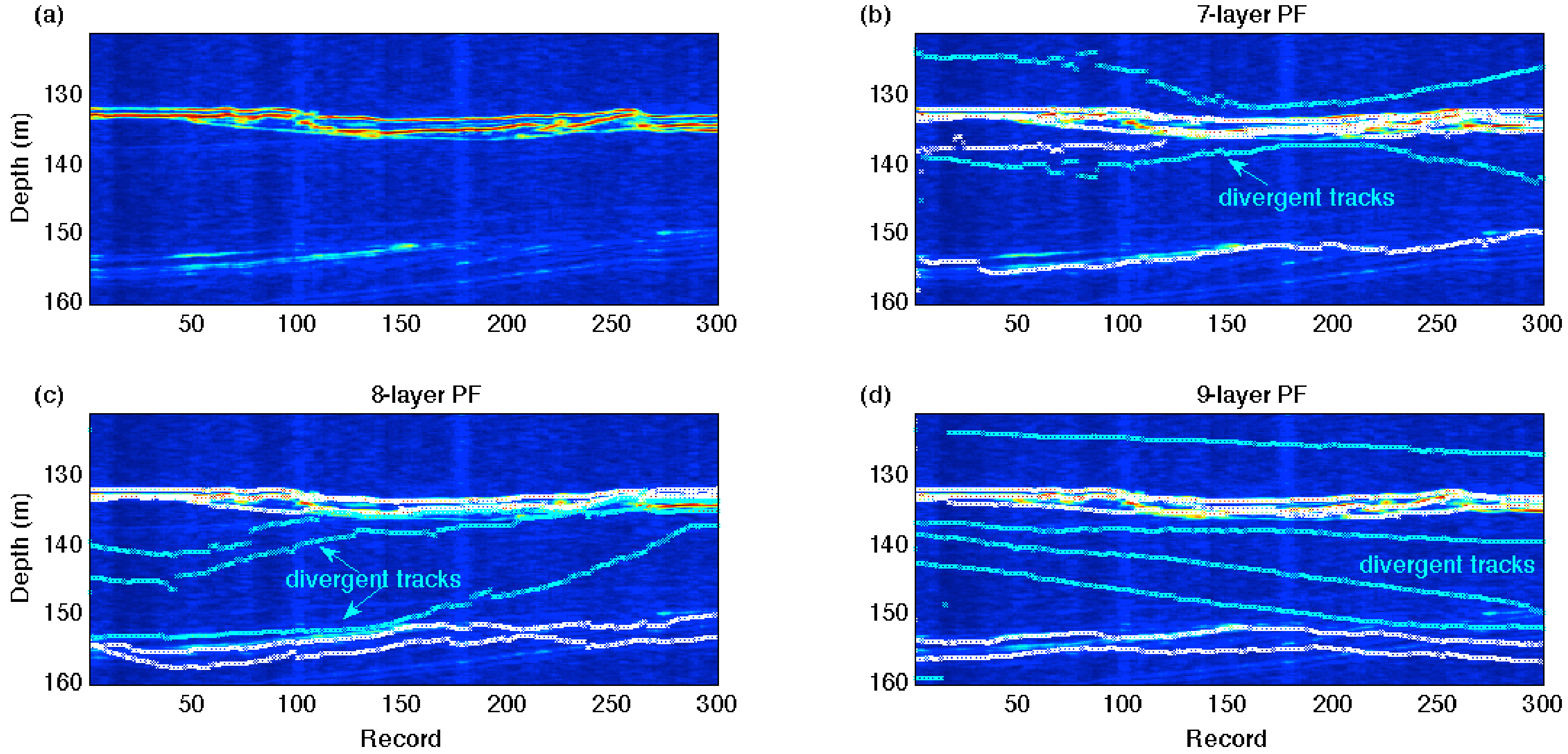




Amplitudes of reflections are related to physical properties of the sea-bottom sediments. Posterior PDFs exhibit interesting multi-modal behavior.

N-layer model – assuming that *N* is known

A simple (single model) PF is employed, assuming that the order is fixed and known.
The presence of seven, eight, and nine reflectors is considered.



Number of reflectors = 8

Number of reflectors = 9

Summarizing model selection in inversion

- Simple model that may fit the data: adequate (and potentially quite accurate) for target detection/localization.

- Fixed layer structure (for example, layer thickness obtained via seismic studies and prior information) – could also provide accurate results for source localization but may provide biased results for geoacoustic properties depending on prior information.

- *N*-layer model when *N* is estimated from data (appropriate modeling favors the smallest values of *N* that provides the best fit; otherwise, models with large *N* would be selected). Not only order can be calculated in this way, but also what sound propagation model is optimal.

- A Markov process model of ocean sediments, Michael Sockell, Ioannis Besieris, Werner Kohler, and Herbert Freese, *J. Acoust. Soc. Am.* 77, 74 (1985)

  An early paper in geoacoustic inversion with stochastic transitioning between sediment models. The problem there was slightly different: estimate *N*-materials rather than layers, but the idea is very similar to what we are currently investigating

Data needs

- Large and closely spaced in time/space measurements allow significant reduction of noise effects and selection of appropriate model. Important factor in improving inversion quality: how to exploit and integrate data to maximize the benefit of inverting and interpreting results.
    - ❖ Combine different data types (propagation and reverberation is one example)
    - ❖ Combine data from different ranges (as an example) with objective/misfit functions and forward models that are more suitable to each separate case.
    - ❖ Compare and interpret inversion results from very similar data sets (parallel closely spaced tracks, for example)

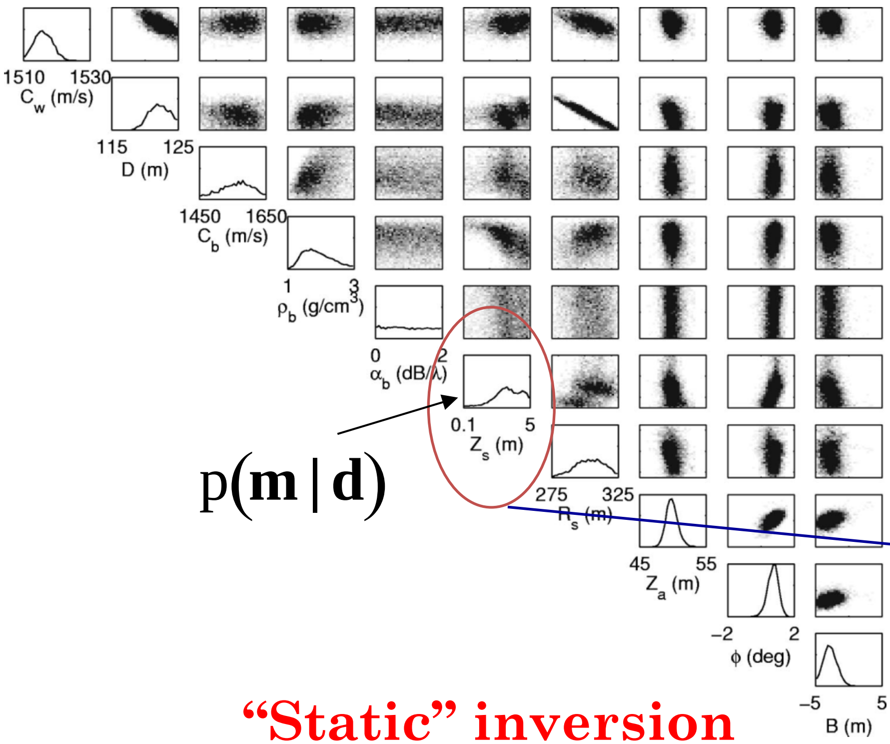
- Can a minimum number of data measurements be determined for a specific inversion task?

Example of data integration:
Static inversion (one frame) vs. Track – and - Invert

Track – and – invert:
Efficient and effective use of data

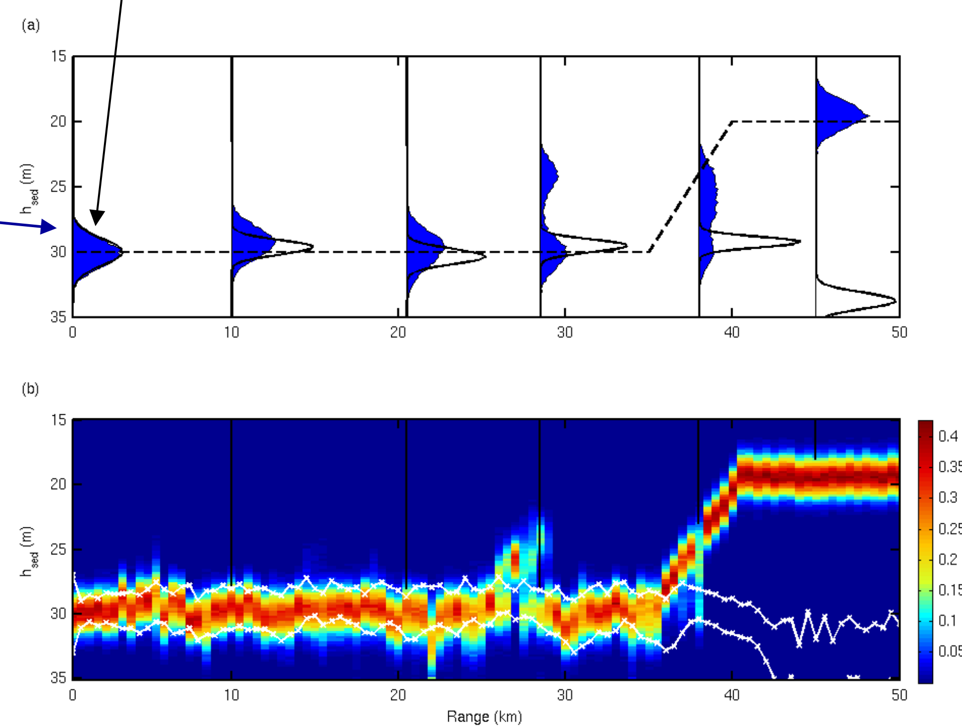
$$p\left(\mathbf{x}_k \mid \mathbf{X}_{k-1}, \mathbf{Y}_k\right)$$

$$\mathbf{X}_{k-1} = \mathbf{x}_{k-1},\ \ldots, \mathbf{x}_0 \qquad \mathbf{Y}_k = \mathbf{y}_k,\ \ldots, \mathbf{y}_0$$



$p(\mathbf{m} \mid \mathbf{d})$

"Static" inversion



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